Ill-posedness of truncated series models of water waves
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A number of approximate models have been introduced for the study of water waves, to avoid the full complexity of the Euler equations. Truncated series models are a family of approximate water wave models which are popular for computing, as they can be treated efficiently by pseudospectral methods. These models are formed by expanding the Dirichlet-to-Neumann operator as a series, truncating the series, and substituting the result into the equations of motion. The full water wave problem is well-known to have a well-posed initial value problem. This leads us to ask whether the truncated series models inherit this feature. We show evidence that the truncated series models of gravity water waves in fact have ill-posed initial value problems. We discuss the case of stronger dispersion; for sufficiently strong dispersion, such as in the case of an elastic bending force, the initial value problem is well-posed. Ill-posedness or well-posedness in the case of surface tension remains an open problem.
This includes joint work with Jerry Bona, Shunlian Liu, David Nicholls, and possibly Michael Siegel.